A Rational Constraint Handling NMPC Method for Systems With Limited Degrees of Freedom
Output Constraints and Limited Actuation

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Motivation

- Real systems often have limited degrees of freedom
  - Many saturated actuators
  - Many measurements
- Real processes have product quality constraints
  - Product must meet quality specifications
  - Error norm minimization not as relevant
- Real processes have safety limitations
  - Bounds on process for safe operation
- Real systems are nonlinear
Proposed Method

- Create prioritized list of control objectives

\[ |e_m(k) - c_i| \leq s_i \ \forall k \]

- Add soft constraint for highest priority objective and solve

\[
\begin{align*}
\min s_i \\
f \left( u^T y^T e^T \right) = 0 \\
A \left[ e^T s_i \right] \leq \pm c_i \\
u_{LB} \leq u \leq u_{UB}
\end{align*}
\]

- Add hard constraint for that objective and repeat

\[
\begin{align*}
\min s_{i+1} \\
f \left( u^T y^T e^T \right) = 0 \\
A \left[ e^T s_{i+1} \right] \leq \pm c_{i+1} \\
u_{LB} \leq u \leq u_{UB} \\
e_{LB} \leq e \leq e_{UB}
\end{align*}
\]

- Use quadratic objective function once possible objectives are met

\[
\min \sum_{k=1}^{P} e(k)^T \Gamma y e(k) + \sum_{k=1}^{M} \Delta u(k)^T \Gamma \Delta u \Delta u(k)
\]
Illustration

Desired Operating Point

Product Quality Limits

$y_1$

$y_2$
Advantages and Disadvantages

- Accommodates qualitative control objectives rationally
- Hard constraints are always feasible
  - Feasible values provided by soft constraint problem
- For $n$ objectives, must solve $n$ NLP problems
  - Still better than mixed-integer optimization using branch-and-bound
  - Always have a viable (possibly sub-optimal) control move
- Not considering guaranteed stability formulation
  - Add terminal state constraint as objective
- Local NLP solves may not find the global solution
  - Global solution may actually be undesirable
  - Can use relaxation methods for bound